

Script for the educational module entitled “The Hendersonian approach: A new way of learning Arterial Blood Gas interpretation”

Title Slide:

This educational video, entitled “The Hendersonian approach: A new way of learning Arterial Blood Gas interpretation”, has been composed by Bob Demers.

Slide 2

This module is a so-called “Interactive Video™”. You will need to view it on a system which provides both audio and video. At various points during the presentation, a chime will sound. At that point, viewers are required to highlight the words or phrases in their handout which correspond to the words/phrases which display in yellow text in the video images. Before we proceed with the video, viewers should ensure that they have a hard copy of the handout, and a highlighter, in hand. If either of these is not available, pause the video at this time in order to secure them.

Slide 3

At the conclusion of this module, viewers will need to: 1) demonstrate their mastery of the Henderson Equation; and 2) use the Henderson-Demers Equation to verify that the parameters embedded within an Arterial Blood Gas, or “ABG”, report, are mutually compatible. Viewers’ acquisition of these skills will be confirmed by the satisfactory completion of a Post-Test, the components of which will be drawn from a test pool comprised of twenty items. If anyone to whom this module has been assigned has already mastered its’ content, s/he will be allowed to “challenge out” of it by satisfactorily completing a Pre-Test comprised of items drawn from the same question pool.

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The staff complements of Intensive Care Units (“ICUs”) are typically comprised of physicians, respiratory care practitioners, and registered nurses. The members of these multidisciplinary teams are obliged to collaborate closely in order to dispense optimal care to their patients. As an integral component of this collaborative practice, they inspect Arterial Blood Gas (or “ABG”) reports carefully and, based on their findings, they develop treatment schemes for their patients. The elements of their interventional therapeutic schemes cover a broad spectrum, ranging from conservative to aggressive. For example, if the patient’s acid-base status is acceptable, but s/he is moderately hypoxemic, appropriate therapy might be as simple, and as conservative, as furnishing the patient with low-flow oxygen by means of a nasal cannula (“nasal prongs”). On the other hand, if a severe acid-base disturbance prevails, the team might be obliged to immediately intubate the patient and institute mechanical ventilation, an admittedly aggressive approach. It is easy to appreciate, then, that the correct interpretation of ABG data is of paramount importance in the context of Intensive Care practice.

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Ever since the ABG analyzer appeared on the clinical scene in the mid-1960’s, clinical educators have been exhorting their students to verify the results of ABG analyses. The vast majority of the time, the data sets resulting from ABG sampling, which emanate from the “black box” which is the blood gas analyzer, are correct. Nevertheless, astute practitioners must always be prepared to confirm that the data upon which they will base important clinical decisions are intrinsically accurate. For that reason, I have been insisting, for decades, that my students be sufficiently well versed in the concepts and principles of acid-base chemistry that they be able to verify the results incorporated in an ABG report before they attempt to implement their treatment strategies.

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As will be discussed shortly, the verification of ABG data formerly required that students refer to handbooks of mathematical tables, an expectation which might be considered unreasonable to impose upon ICU practitioners. Happily, though, the medical landscape has changed dramatically during the past few years, and digital computers have proliferated to such a degree that it is now uncommon to find an ICU bed which lacks access to a dedicated, networked computer terminal. Within this clinical/workflow environment, new opportunities for diagnosis and management have emerged. In this presentation I hope to render the mathematical description of a chemical equilibrium system less complex, and to exploit the functionalities of computers to convey a more visceral and intuitive grasp of acid-base physiology than has been available in the past.

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Physicians, Clinical Chemists, and RCPs typically encounter the concepts of acid-base chemistry and the physiology of equilibrium systems quite early in their training. Of the various parameters embedded within an ABG report, three are specifically applicable to acid-base balance: the arterial pH, symbolized as “pH_a”; the arterial carbon dioxide tension, symbolized as “p_aCO₂”; and the bicarbonate ion concentration, symbolized as [HCO₃]. The interrelationship among these variables is precisely regulated in a physiologic, mathematical, and chemical sense by means of an equation which is familiar to students everywhere.

Slide 8

The equation to which I refer is the Henderson-Hasselbalch Equation, shown here. Unfortunately, this equation is hugely, and universally, unpopular with students. The intense distaste which learners vocalize for the Henderson-Hasselbalch Equation has served as a stumbling-block to instructors and clinical preceptors ever since Karl Albert Hasselbalch published it about a century ago.

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It is not widely appreciated that Lawrence J. Henderson developed the equation that bears his name fully eight years before Professor Hasselbalch derived the Henderson-Hasselbalch Equation.

The Henderson Equation quantifies hydrogen ion concentration directly, in units of “nanomoles” (or billionths of a mole) per Liter. In other words, the Henderson Equation does not invoke the concept of the logarithm, which renders it intrinsically simpler, and more straightforward, than the Henderson-Hasselbalch Equation which succeeded it. There is a very good reason why Henderson did not incorporate a “pH” term in his equation: Søren Sørensen did not develop the concept of pH until the year following the publication of Henderson’s paper!

Slide 10

Students of fundamental chemistry learn that any compound which bears the label of “acid” is categorized as such because of its’ ability to donate hydrogen ions to a solution. If an acid dissociates completely, it will serve as a rich source of these ions, and will be characterized as a “strong acid”. Hydrochloric acid, sulfuric acid, and nitric acid provide examples of strong acids. On the other hand, if the compound is observed to dissociate only weakly into its’ constituent ions, most of it will remain in its’ associated state, and it is termed a “weak acid”. Carbonic acid and acetic acid furnish examples of weak acids. But, whether an acid is strong or weak, all acids exhibit a propensity for donating hydrogen ions to a solution, to a greater or lesser extent. Stated another way, the hydrogen ion is the single entity among all chemical species which is responsible for acid behavior. Because of this, many consider it far more intuitive to quantify the acidity of a solution in terms of its’ hydrogen ion concentration, in lieu of citing its’ pH. Sørensen defined pH as “the negative logarithm of the hydrogen ion concentration”, a concept which is somewhat abstruse and nebulous, even to chemists.

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The Hendersonian approach will, admittedly, be completely unfamiliar to most viewers of this presentation. Nevertheless, by virtue of the experience that they’ve acquired with respect to the conventional method of ABG interpretation, most practitioners will already be familiar with the normal ranges for $p_a\text{CO}_2$ and $[\text{HCO}_3^-]$ listed here. The only additional information needed is the homeostatic range for $[\text{H}^+]$, which is 35 to 45 nanomoles per Liter, inclusive.

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The carbon dioxide tension is used to determine the status of the so-called “respiratory” component. The elimination of that endogenously-produced gas is modulated, of course, by the lungs, and CO_2 constitutes an acid precursor, insofar as its’ chemical combination with water generates carbonic acid, H_2CO_3 . If the prevailing $p_a\text{CO}_2$ is low, owing to hyperventilation, this represents a deficit in that acid precursor, and will tend to elicit an alkalemia. And, because the mechanism here is referable to the respiratory component, the disorder will classify as a “respiratory alkalemia”. Conversely, a hypoventilatory state, sufficient to elevate the $p_a\text{CO}_2$ to a level in excess of the upper limit of its’ normal range, constitutes an acid surplus, and will be characterized as a “respiratory acidemia”.

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The bicarbonate ion concentration is used to determine the status of the so-called “metabolic” component. The elimination of that ion is modulated by the kidneys, and HCO_3^- constitutes a fairly strong base, insofar as it readily associates, or combines, with hydrogen ion, effectively removing it from solution. If the prevailing $[\text{HCO}_3^-]$ is low, a base deficit exists, which will tend to elicit an acidemia. And, because the mechanism here is referable to the metabolic component, the disorder will classify as a “metabolic acidemia”. Conversely, an $[\text{HCO}_3^-]$ in excess of the upper limit of its’ normal range constitutes a base surplus, and will be characterized as a “metabolic alkalemia”.

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For the past few years, I have employed, with some success, the Henderson Equation as an educational strategy to teach the intricacies of ABG interpretation. In that connection, a Worksheet has been developed to facilitate the application of the equation to actual ABG results. Viewers of this module are encouraged to inspect the full-page copy of this Worksheet incorporated in the handout; the handout is downloadable from the same site from which this video was accessed. In order to have the time to examine the Worksheet carefully, it is advisable to hit the space bar on the computer keyboard in order to pause the video. If you are viewing this module on an alternative device, such as a tablet computer, you’ll need to employ the Movie Controller in order to pause the video.

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Notice that the user enters the two-digit number, corresponding to the arterial carbon dioxide tension, in torr, listed in the ABG report in the blue rectangular box. Depending on whether this value is below, within, or above the normal range, the user determines if a respiratory alkalemia, a respiratory homeostasis, or a respiratory acidemia exists, respectively. The user then enters the two-digit value for bicarbonate ion concentration, in milliequivalents per Liter, in the red rectangular box. A value which lies below, within, or above the homeostatic range allows the user to characterize the patient’s metabolic status as a metabolic acidemia, a metabolic homeostasis, or a metabolic alkalemia, respectively. The next step of the process consists of implementing the Henderson Equation in order to generate a figure for the hydrogen ion concentration, in nanomoles per Liter.

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The numerical value of the result will represent an alkalemic, a homeostatic, or an acidemic state, corresponding with either a subnormal, a normal, or a supernormal value for $[\text{H}^+]$. Suppose the $[\text{H}^+]$ is less than 35 nM/L, the lower limit of its’ normal range. By definition, this is an acid deficit, or alkalemia, which has been created by a low value for the ratio between $p_a\text{CO}_2$ and $[\text{HCO}_3^-]$. Arithmetically, this result could be attributable to either a small numerator (a respiratory alkalemia) or a large denominator (a metabolic alkalemia). In contradistinction to this situation, if the numerical value of the $[\text{H}^+]$ exceeds the upper limit of its’ normal range, the ratio must be commensurately large, which will have been triggered either by the presence of a large numerator (a respiratory acidemia) or a small denominator (a metabolic acidemia).

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At this stage of the process, the user is able to identify which component, respiratory or metabolic, is tending in the same acid-base direction as the $[H^+]$. If it is observed that each of them is tending in the same direction, so as to reinforce, rather than oppose, each other, the acid-base disorder is characterized as “mixed”-- either a mixed alkalemia or a mixed acidemia. More commonly, however, the respiratory and the metabolic components will be opposing each other, and the one which is tending in the same acid-base direction as the $[H^+]$ will be identified as the primary component. The compensatory component will be identified, by default, as the remaining component.

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If the prevailing value of the compensatory component is seen to be lying within its' homeostatic range, the dysfunction will be characterized as an “uncompensated” disorder. If the value lies outside its' homeostatic range, but has not been successful in restoring the $[H^+]$ value to its' homeostatic range, the disorder will be classified as a “partially compensated” dysfunction. On the other hand, if the prevailing value of the compensatory parameter is both outside of its' homeostatic range, and has succeeded in restoring the $[H^+]$ to the homeostatic range, the disorder will classify as a “completely compensated” dysfunction.

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Finally, the user will synthesize the steps described above and will proceed to fully classify the prevailing acid-base disorder as: “uncompensated” or “partially compensated” or “completely compensated”; “respiratory” or “metabolic” or “mixed”; “acidemia” or “alkalemia”. Of course, it's possible that all of the acid-base parameters will lie within their respective homeostatic ranges, which will prompt the clinician to characterize the patient's condition as “homeostatic”. The process outlined in the Henderson Equation Worksheet involves one additional step which enables users to verify the validity of the ABG report, and that step will be described shortly.

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Consider this scenario, which pertains to a patient who is suffering from chronic obstructive pulmonary disease (COPD) of moderate severity. He presents with a p_aCO_2 of 62 torr and an $[HCO_3^-]$ of 27 mEq/L. Substitution of these values into the Henderson Equation yields an $[H^+]$ of $(\{24 \cdot 62\} / 27 =)$ 55 nM/L. Notice that the elevated $[H^+]$ here has resulted from an abnormally high numerator (a respiratory acidemia). The denominator is observed to be marginally higher than the upper limit of its' normal range (a very mild metabolic alkalemia), which is precisely the type of metabolic compensation that we would anticipate in the face of a primary respiratory acidemia. We would completely and concisely characterize this acid-base dysfunction as a partially compensated respiratory acidemia.

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Next, we’ll consider the case of a toddler who was fed a strong alkaline solution (lye) by his older brother in the course of “playing house” without parental supervision. An arterial blood specimen obtained upon the patient’s arrival in the Emergency Department reveals a $p_a\text{CO}_2$ of 48 torr and an $[\text{HCO}_3^-]$ of 37 mEq/L. Substitution of these values into the Henderson Equation generates an $[\text{H}^+]$ value which is subnormal: $[\text{H}^+] = (\{24 \cdot 48\} / 37 =) 31 \text{ nM/L}$. The numerator cannot be blamed for the abnormal result, insofar as it is observed to be slightly higher than the upper limit of its’ normal range, which would tend to make the ratio higher, and not lower. The only way that the quotient can be low is for the denominator to be substantially elevated, which is, in fact, the case. Therefore, this acid-base disturbance is revealed as being a partially compensated metabolic alkalemia, the chemical consequence of the ingestion of the strongly caustic and alkaline solution, sodium hydroxide.

Slide 22

As devoted an advocate of the Henderson Equation as I have come to be, I must readily concede that, whether we like it or not, the custom of expressing acidity by citing pH is solidly entrenched in the medical/allied health literature. Stated another way, if we were to inform an attending physician that her/his patient were exhibiting an $[\text{H}^+]$ of, say, 55 nM/L, s/he would probably have no idea what we were talking about! So, if we hope to communicate coherently with the other members of the healthcare team, we must devise a method whereby we can convert any given $[\text{H}^+]$ value to its’ corresponding pH reading. The Henderson-Demers Equation provides us with that method.

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This equation, the derivation of which is incorporated in the script for this video, reads as follows:

$$pH = 9.00 - \log_{10} [H^+]_L$$

where, once again, the subscript “L” has been applied to indicate that the hydrogen ion concentration in this expression is in units of nanomoles per Liter, and not moles per Liter. The format of this equation resembles that of the Henderson-Hasselbalch Equation, and it could be considered to represent a somewhat simpler alternative to that expression.

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In this slide, the Henderson-Hasselbalch and the Henderson-Demers Equations are displayed side-by-side for comparison purposes. The implementation of either expression requires that the user multiply the numerical value of $p_a\text{CO}_2$, either by a non-integer (0.03) or an integer (24). Next, the user must undertake a division process, and, finally, s/he must determine the logarithm of the result.

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The direct comparison conducted in the previous slide reveals that the Henderson-Demers Equation is easier to implement than the Henderson-Hasselbalch Equation, but only to a slight degree, if the user’s goal is to determine pH in one continuous process. However, those who implement the Henderson Equation Worksheet will determine the $[H^+]$ in nanomoles per Liter as a preliminary exercise, in order to determine the overall acid-base status of the patient.

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In doing so, they will generate a two-digit number for the $[H^+]$, and, once this number is in hand, the subsequent determination of pH will be child’s play. For example, recall that the patient described in Scenario 1 exhibited an $[H^+]$ value of 55 nM/L. The determination of the corresponding pH here is simply 9.00 minus the logarithm of 55, or 7.26 units. Similarly, the pediatric patient described in Scenario 2 manifested an $[H^+]$ level of 31 nM/L, allowing us to easily determine the corresponding pH value of 7.51 units.

Slide 27

Not so long ago, clinicians would have been forced to sequester handbooks incorporating logarithm tables within the pockets of their lab coats in order to be able to make the calculations outlined in the previous slide. Happily, though, the advent of the Digital Revolution has freed us from that drudgery. The “Calculator” applications (“apps”) which are an integral component of virtually every smartPhone, allow us to perform the requisite computations at the stroke of a key. In fact, the “twenty-somethings” who, for the most part, comprise the student bodies of respiratory therapy, medical, and nursing schools, are children of the Digital Age. They reflexly reach for their smartPhone when required to perform even the simplest mathematical operations, such as solving the Henderson Equation. The widespread availability of this technology renders it eminently reasonable to insist that students, and the ICU Team members to whom they might be assigned for a clinical rotation, verify each and every ABG report issued for patients under their care. Once it has been confirmed that the parameters supplied by the lab obey the Henderson Equation, the student/clinician can proceed to implement an interventional therapeutic strategy with confidence.

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Many brands/models of smartPhone incorporate a calculator as an integral feature. In most cases, when this app is launched, a simple calculator is displayed on the face of the device. Much like a typical adding machine, these calculators enable users to perform simple tasks, such as addition, subtraction, multiplication, or division. But they lack the ability of so-called “scientific calculators” to undertake exponential operations, such as computing a square root, or determining a logarithm. In the next slide, we will demonstrate how a particular brand/model of smartPhone, Apple’s iPhone® (Apple, Inc., Cupertino, CA), can be employed to convert a simple calculator to a scientific calculator.

Slide 29

No narrative

Slide 30

In this and the next three slides, viewers are advised to manually enter numerical values in each of four Henderson Equation Worksheets embedded in their handouts in order to observe how this tool can be employed as a learning resource. For these cases, ABG values which were observed in four actual patients will be used. The first of these was a thirty-four-year-old male who was transported to the Emergency Department following an overdose of barbiturates. His ABG data read as shown here. When the Henderson Equation is applied to these parameters, the patient’s $[H^+]$ is calculated to be 55 nM/L, and he is revealed to be suffering from an uncompensated respiratory acidemia. The observation of an $[HCO_3^-]$ value within its’ homeostatic range suggests that the hypoventilatory state is acute, rather than chronic, in accordance with the patient’s clinical history. This patient was intubated and mechanically ventilated without delay, which is appropriate in the face of an acute hypercapnic episode. The implementation of the Henderson-Demers Equation generated a pH figure of 7.26, documenting that the ABG data reported by the lab were compatible with the Henderson and the Henderson-Hasselbalch Equations.

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An eight-year-old male, admitted to Mattel Children’s Hospital, presented with these ABG values during air breathing. After a three-hour period during which Bilevel Positive Airway Pressure (BiPAP) was tried, the ABG report shown here was obtained. Substitution of these p_aCO_2 and $[HCO_3^-]$ values into the Henderson Equation reveals an $[H^+]$ of 53 nM/L, which is consistent with a moderate acidemia. The patient’s acid-base status is revealed as an uncompensated respiratory acidemia. Here again, the presence of an uncompensated respiratory disorder suggests the presence of an acute, and not a chronic, respiratory condition. At this point in his course, he was intubated, and ABG values were returned to their normal ranges within one-half hour. Ultimately, a diagnosis of propionic acidemia was made. The implementation of the Henderson-Demers Equation generated a pH figure of 7.28, which served to verify that the ABG data reported by the lab were valid.

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These ABG data were obtained from a three-month-old infant who had resided in the Neonatal Intensive Care Unit since birth. On the day of NICU discharge, she manifested a completely compensated respiratory acidemia secondary to bronchopulmonary dysplasia (BPD), which had been triggered by prolonged mechanical ventilation at elevated $F_{I}O_2$ s. As can be seen here, her primary respiratory acidemia was being well compensated by a supervening metabolic alkalemia. This picture is typical of clinically stable BPD patients whose kidneys are afforded the requisite time to compensate for the inciting hypercapnic state. She was discharged on low-flow oxygen delivered by nasal cannula. Substitution of the prevailing p_aCO_2 and $[HCO_3^-]$ values into the Henderson Equation yield an $[H^+]$ reading of 44 nM/L and a corresponding pH of 7.36, which confirms the accuracy of the lab-generated ABG values.

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Three weeks after discharge, the same baby was brought to the Emergency Department in respiratory distress, at which point the ABG results shown here were obtained. Application of the Henderson Equation Worksheet leads us to conclude that the patient is exhibiting a partially compensated respiratory acidemia. Her $[H^+]$ is now 49 nM/L, and her pH is 7.31, in perfect agreement with the lab-generated value, which reinforces the reliability of the ABG report. Actually, since we are aware that this patient’s antecedent acid-base condition was a completely compensated respiratory acidemia, we are able to conclude that she has sustained an episode of acute-on-chronic hypercapnia. An elevated white blood count and the presence of an infiltrate on chest X-ray confirmed a diagnosis of bacterial pneumonia, and she was successfully treated with a course of intravenous antibiotics. Fortunately, intubation and mechanical ventilation were averted in this case.

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The members of the ICU Team collaborate to create interventional therapeutic strategies which are often quite aggressive. This renders it mandatory that they be absolutely sure that the ABG interpretations which they formulate be unimpeachably accurate and which, in turn, obliges them to be intimately conversant with the concepts of acid-base chemistry. Traditionally, the mastery of these principles required practitioners to be facile with the Henderson-Hasselbalch Equation, notwithstanding the fact that this expression is intrinsically complex. In the final analysis, it is imperative that every member of the ICU Team incorporate ABG interpretation into his/her clinical skills inventory.

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Clinicians are not unique in regarding the Henderson-Hasselbalch Equation as nebulous and arcane. The Henderson Equation, which was actually a forerunner of the Henderson-Hasselbalch Equation, is both more straightforward and more intuitive than the latter expression, which was published eight years later. The Henderson Equation Worksheet was developed in order to guide junior members of the healthcare team through a series of steps which culminate in the formulation of a classification of any acid-base disorder that might be encountered clinically.

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When a user implements the Worksheet, s/he will ultimately be enabled to characterize the acid-base disorder present, if any, as an: “uncompensated” or “partially compensated” or “completely compensated”; “respiratory” or “metabolic” or “mixed”; “acidemia” or “alkalemia”.

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The final step embedded in the Worksheet allows the user to determine the prevailing pH by applying the Henderson-Demers Equation. When the pH derived by the implementation of that expression is observed to coincide with the pH value furnished by the lab, the validity of those data are confirmed, and the ICU Team can apply their therapeutic strategy with confidence. It has been

observed that assigning a small series of data sets to students previously unfamiliar with the process of ABG interpretation familiarizes them with the analytic sequence embodied within the Worksheet, as they work through the process in repetitive fashion. Within a very short timeframe, they are able to undertake this task in the absence of any written adjunct whatsoever.

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Until recently, one of the factors impairing our ability to determine pH at the bedside resided in the requirement that practitioners have ongoing access to Tables of Logarithms. This, in turn, obliged clinicians to tote around hard-copy manuals, an admittedly cumbersome chore. But the advent of the Digital Revolution has liberated us from that burdensome and onerous state, inasmuch as voluminous tabular data is, quite literally, always at our fingertips! Consequently, all practitioners, from the rankest neophyte to the most seasoned veteran, can master the intricacies of this crucially important clinical competency. Many of the students who have mastered ABG interpretation by means of the Hendersonian approach have been prompted to declare that the experience has been downright “fun” for them! This is certainly a far cry from the highly unfavorable response that most students formerly vocalized when the rigorous Henderson-Hasselbalch paradigm was used as a learning tool.

Slide 39

No narrative.

References:

1. Hasselbalch KA. Die Berechnung der Wasserstoffzahl des Blutes aus der freien und gebundenen Kohlensäure desselben, und die Sauerstoffbindung des Blutes als Funktion der Wasserstoffzahl. **Biochem Zeitschr** 1916; 78: 112–144.
2. Henderson LJ. The theory of neutrality regulation in the animal organism. **Am J Physiol** 1908; 21: 427-448.
3. Sørensen SPL. Über die Messung und die Bedeutung der Wasserstoffionenkoncentration bei enzymatischen Prozessen. **Biochem Zeitschr** 1909; 21: 131-304.

Appendix 1:

Derivation of the Henderson-Demers Equation:

The Henderson Equation reads as follows:

$$[H^+]_L = 24 \cdot \left(\frac{p_a CO_2}{[HCO_3^-]} \right)$$

where “[H⁺]_L” represents the hydrogen ion concentration expressed in nanomoles per Liter.

[H⁺], the hydrogen ion concentration expressed in moles per Liter, is many (one billion!) orders of magnitude smaller than [H⁺]_L; specifically: [H⁺] = [H⁺]_L • 10⁻⁹.

Hence, in order to convert [H⁺]_L to [H⁺], we are obliged to multiply by 10⁻⁹:

$$[H^+] = 24 \cdot \left(\frac{p_a CO_2}{[HCO_3^-]} \right) \cdot 10^{-9}$$

We now proceed to take the logarithm of each term, which, by applying the rules of logs, yields:

$$\log_{10} [H^+] = \log_{10} \left\{ 24 \cdot \left(\frac{p_a CO_2}{[HCO_3^-]} \right) \right\} + \log_{10} 10^{-9}$$

which reverts to the following form:

$$\log_{10} [H^+] = \log_{10} \left\{ 24 \cdot \left(\frac{p_a CO_2}{[HCO_3^-]} \right) \right\} - 9.00$$

Multiplying each term by negative one yields:

$$-\log_{10}[H^+] = 9.00 - \log_{10} \left\{ 24 \cdot \left(\frac{p_aCO_2}{[HCO_3^-]} \right) \right\}$$

Since “pH” is defined as “the negative logarithm of the hydrogen ion concentration”, this equation reverts to:

$$pH = 9.00 - \log_{10} \left\{ 24 \cdot \left(\frac{p_aCO_2}{[HCO_3^-]} \right) \right\}$$

Notice that a portion of the second term on the right side of the equation is enclosed within fluted brackets. That portion of the expression corresponds to the value of $[H^+]_L$, the hydrogen ion concentration value in nanomoles per Liter, as given by the Henderson Equation. We substitute for that entity in order to obtain the final form of the Henderson-Demers Equation:

$$pH = 9.00 - \log_{10}[H^+]_L$$

Appendix 2:

Derivation of the Simplified Henderson-Demers Equation:

Kristen Merriman, MS, RRT, RCP, a respiratory therapist who is particularly conversant in mathematics, pointed out that the Henderson-Demers Equation is subject to further simplification as follows:

$$pH = 9.00 - \log_{10} \left\{ 24 \cdot \left(\frac{p_aCO_2}{[HCO_3^-]} \right) \right\}$$

This expression can be expanded, by separation of terms, to read as follows:

$$pH = 9.00 - \log_{10} 24 - \log_{10} \left(\frac{p_aCO_2}{[HCO_3^-]} \right)$$

which, in turn, can be simplified by subtracting the logarithm of 24 (1.38) from 9.00:

$$pH = 7.62 - \log_{10} \left(\frac{p_aCO_2}{[HCO_3^-]} \right)$$

This is the final form of the Simplified Henderson-Demers Equation. Clinicians who are equipped with any hand-held computer or smartPhone which incorporates a scientific calculator can employ this equation at the bedside to quickly and easily verify that the pH_a , p_aCO_2 , and $[HCO_3^-]$ values contained within any ABG report are mutually compatible with both the Henderson-Hasselbalch and the Henderson Equations.